Technical Notes

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On Prediction of Separated Boundary Layers with Pressure Distribution Specified

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Nomenclature

C_f = skin-friction coefficient
C_D = dissipation integral $C_D \equiv 2/\rho_e u_e^3 \int_0^\infty \tau(\partial u/\partial y)\,dy$
f = Mach number—shape factor function

$$f \equiv \frac{3\gamma-1}{\gamma-1} + \left[2 + \frac{\gamma+1}{\gamma-1}\left(\frac{m_e}{1+m_e}\right)\right]H + \left[\frac{M_e^3}{m_e(1+m_e)}\right]Z$$

H = boundary-layer parameter, $H \equiv \theta/\delta^*$
J = boundary-layer parameter, $J = \delta_E/\delta^*$
M_e = edge Mach number
m_e = $(\gamma-1)M_e^2/2$
u, v = velocity components
X = coordinate in flow direction
Z = boundary-layer parameter, $Z \equiv (\delta-\delta^*)/\delta^*$
δ = boundary-layer thickness
δ_E = boundary-layer kinetic energy thickness
δ^* = boundary-layer displacement thickness
θ = boundary-layer momentum thickness
Θ = flow angle at edge of viscous region $\Theta \equiv \tan^{-1}(v_e/u_e)$
τ = shear stress

WHEN a boundary layer, either laminar or turbulent, separates, prediction of the resulting flowfield by classical boundary-layer methods fails. The reason for this failure is that the viscous and inviscid regions of the flowfield are no longer independent; the pressure distribution is determined by the interaction between these two regions. This is known as the strong interaction problem. The results of Lees, Reeves, Alber, and Hunter[1-4] seem to indicate that, in a great many cases, the boundary-layer approximations to the viscous flow equations are valid.

The abovementioned comments notwithstanding, it may sometimes be desirable to calculate separated boundary-layer behavior from a specified pressure distribution. Two such cases that come to mind are the testing of a proposed viscous layer model with data in which the separated region pressure distribution has been measured[5] and the problem of the complete calculation of a subsonic separated flowfield, which must presumably be done in an iterative manner, alternating between viscous and inviscid calculations. A widely accepted method of calculating separated boundary layers makes use of the integral momentum and mechanical energy equations.[1-4] It is the purpose of this Note to demonstrate that direct solution of these equations with pressure distribution specified is not always feasible and to suggest a proper method of obtaining such solutions.

The integral momentum and mechanical energy equations in the form applied to separated boundary layers are[1-4]

Received September 10, 1973; revision received April 17, 1974. This paper was written while the author was a Summer Faculty Fellow at NASA Lewis Research Center. The author is also indebted to I. E. Alber of TRW Systems for clarification of some points presented.
Index category: Jets, Wakes, and Viscid-Inviscid Flow Interactions.
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$$H\frac{d\delta^*}{dx} + \delta^*\frac{dH}{dx} + [2H+1]\frac{\delta^*}{M_e}\frac{dM_e}{dx} = \frac{C_f}{2} \tag{1}$$

$$J\frac{d\delta^*}{dx} + \delta^*\frac{dJ}{dx} + 3J\frac{\delta^*}{M_e}\frac{dM_e}{dx} = C_D \tag{2}$$

In these equations it is assumed that compressibility effects have been eliminated by a Stewartson-type transformation. If some velocity profile family is assumed such that $J = J(H)$ and appropriate "laws" are introduced for C_f and C_D, Eqs. (1) and (2) become two equations in the two unknowns δ^* and H if $M_e(x)$ is known.

Using Eq. (1), Eq. (2) may be written

$$\delta^*\left[\frac{dJ}{dH} - \frac{J}{H}\right]\frac{dH}{dx} = C_D - \frac{JC_f}{2H} + \left(\frac{1}{H} - J\right)\frac{\delta^*}{M_e}\frac{dM_e}{dx} \tag{3}$$

Equation (3) governs the shape factor H and may be solved, provided that $dJ/dH - J/H \neq 0$. If the velocity profile family selected for the analysis has a point on the $J-H$ curve such that tangent to the curve passes through the origin, solutions of Eqs. (1) and (2) are not possible in the neighborhood of such a point. Such points have been labeled "velocity profile critical points" by Shamroth.[6] The existence of such points is not a remote possibility; the present author has located such points in a great many of the velocity profile families commonly used in separated flow studies. A graphical method of locating such points is shown for the "mean flow" theory similar reverse flow profiles of Alber[7] in Fig. 1. If a polynomial curve fit is employed for a particular profile family, the location of such points is dependent on the particular curve fit. Table 1 summarizes location of $H-J$ critical points for several commonly used velocity profile families.[1-3,7]

The failure of Eq. (3) to produce a solution in the neighborhood of an $H-J$ critical point should not be interpreted as a failure of the set of Eqs. (1) and (2) to represent the phenomenon under consideration. The proper interpretation should be as follows. At some point in the flowfield, the left-hand side of Eq. (3) goes to zero. In order to maintain consistency, the right-hand side of Eq. (3) should also go to zero. Because of approximations

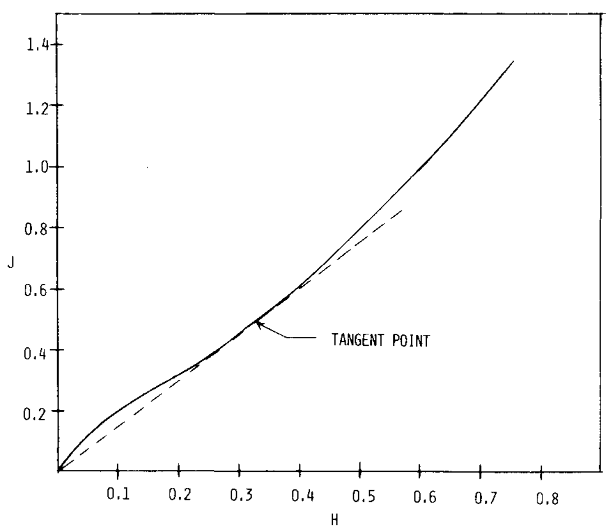

Fig. 1 Location of $H-J$ critical point for mean-flow profile family of Alber.[7]

Table 1 $H-J$ critical points for various velocity profile families

Profile family	Author	$H-J$ Critical point(s)
Falkner-Skan	Lees-Reeves[2]	$H = 0.2458, 0.2295$ 0.2237
Falkner-Skan	Lees-Reeves[2]	$H = 0.4570$
Turbulent similarity mean flow theory	Alber[7]	$H = 0.3175$
Turbulent similarity energy theory	Alber[7]	$H = 0.3250$

in assuming simultaneously an $H-J$ relation, a C_f model, and a C_D model, it is highly unlikely that both sides of Eq. (3) go to zero at the same time. Since $M_e(x)$ is presumed known, the $H-J$ relationship and/or the C_f model and/or the C_D model would have to be adjusted in an arbitrary manner in order to force both sides of Eq. (3) to zero at the same time. This is clearly distasteful. If $M_e(x)$ is the current distribution in an iteration scheme, the calculated $M_e(x)$ is presumably incorrect at intermediate steps and there would be no justification for adjusting the equations based on this wrong value. On the other hand, if $M_e(x)$ is known from measurements, it represents information of a higher degree of accuracy than is available for the $H-J$, C_f, and C_D models, which being only approximations, probably should not be expected to reproduce the exact flow in such a manner that both sides of Eq. (3) go to zero simultaneously.

The way out of this dilemma is to treat the problem as a strong interaction, with $M_e(x)$ unknown. The integrated continuity equation is introduced[1-4]

$$\left[H + \left(\frac{1+M_e}{m_e}\right)\right]\frac{d\delta^*}{dx} + \delta^*\frac{dH}{dx} + f\frac{\delta^*}{M_e}\frac{dM_e}{dx} = \frac{\tan\Theta}{m_e} \quad (4)$$

Solving Eqs. (1, 2, and 4) by Cramer's Rule

$$(\delta^*/M_e)\,dM_e/dx = N_1/D \quad (5)$$

$$\delta^*\,dH/dx = N_2/D \quad (6)$$

$$d\delta^*/dx = N_3/D \quad (7)$$

The forms of the N's and D are given elsewhere.[1-4] A singularity would occur in the preceding set of equations at $D = 0$. In this set, however, D is a function not only of the profile shape parameters H, J, and Z, but also of the external Mach number M_e. Apparently singularities in the preceding set only occur if M_e is greater than approximately 1.5. This is the well-known Crocco-Lees singularity and has actual physical significance.[1,8] Its appearance is not related to the $H-J$ critical point encountered previously.[6]

Since the set of Eqs. (5–7) does not contain the $H-J$ critical point, it is desirable to solve this set. This is done by prescribing Θ and solving the set. In an interaction calculation, the resulting solution is used in an inviscid calculation to obtain a better estimate for Θ. If the actual pressure (M_e) distribution is known, a trial and error solution is suggested. The Θ distribution is adjusted until the calculated $M_e(x)$ agrees with the measured value. It should be noted that this method is similar to that of Kuhn and Nielsen,[9] in which the skin friction is arbitrarily specified and $M_e(x)$ calculated from the boundary-layer equations. The main advantage of the approach outlined previously over that of Kuhn and Nielsen is that in a complete interaction calculation, an assumed Θ distribution can be continually updated from an inviscid analysis of the outer flow, while an assumed C_f distribution must be adjusted in some completely arbitrary manner.

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Hot-Wire Measurements in a Supersonic Jet at Low Reynolds Numbers

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Introduction

THERE is a growing community of researchers who believe that the turbulence in a jet has a degree of orderly structure whose characteristics are determined by the instability process. In the case of the supersonic jet, Tam[1] and Bishop et al.,[2] have theorized that the large-scale instability process is fundamental to the turbulent structure which contributes most substantially to the noise radiation from the jet. In fact, Tam[1] has developed an interesting theory which demonstrates how certain disturbance wavelengths are selectively amplified. Since in Tam's theory all nonlinear terms in the disturbance equations are neglected, it is strictly only applicable in laminar jets. The large velocity fluctuations found in the classical turbulent jet would normally preclude using linear stability theory in this flow situation, particularly if a high degree of accuracy is desirable. However, Landahl[3] and Crow and Champagne[4] have demonstrated that the modes of the linear stability problem are useful in describing the fluctuations in turbulent shear flows.

The theoretical analyses of Tam[1] and other supersonic jet noise researchers need experimental evidence against which to check their predictions. The particular need is information about the noise producing flow fluctuations within the jet. Some experimenters have used schlieren or holograph techniques to visualize these disturbances.[5,6] Our approach differs from theirs not only in the measurement technique but also in the fact that ours focuses on the classical laminar instabilities. By exhausting the jet into a vacuum chamber, low enough Reynolds numbers are obtained so that there are a few diameters of laminar flow in the jet.

Received November 6, 1973; revision received March 11, 1974. This research was supported by the National Science Foundation under Grant GK-32686. The authors would like to acknowledge the assistance of G. L. Morrison and T. R. Troutt.

Index categories: Aircraft Noise, Aerodynamics (Including Sonic Boom); Jets, Wakes, and Viscid-Inviscid Flow Interactions.

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